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Kaon Condensation in Dense Stellar Matter *

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Abstract

This article combines two talks given by the authors and is based on works done in collaboration with G.E. Brown and D.-P. Min on kaon condensation in dense baryonic medium treated in chiral perturbation theory using heavy-baryon formalism. It contains, in addition to what was recently published, astrophysical backgrounds for kaon condensation discussed by Brown and Bethe, a discussion on a renormalization-group analysis to meson condensation worked out together with H.K. Lee and S.-J. Sin, and the recent results of K. M. Westerbergh in the bound-state approach to the Skyrme model. Negatively charged kaons are predicted to condense at a critical density $2 \lesssim \rho/\rho_0 \lesssim 4$, in the range to allow the intriguing new phenomena predicted by Brown and Bethe to take place in compact star matter.

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1 Motivation

Recent work by Bethe and Brown[1] on the maximum mass of stable compact stars – called “neutron stars” in the past but more appropriately “nuclear (or nucleon) stars” – suggest that the nuclear equation of state (EOS) in the interior of compact stars must be considerably softened at densities a few times the nuclear matter density ρ_0 by one or several hadronic phase transitions. It is now fairly clear that neither pion condensation nor quark matter will figure at a density low enough to be relevant to the star matter although the issue is not yet completely settled. As Bethe and Brown suggest, kaon condensation could however take place at a density 3–4 times the normal matter density and hence play an important role in explaining the remarkably narrow range of compact star masses observed in nature[2].

The aim of this talk is to describe a higher-order chiral perturbation calculation that predicts the critical density for kaon condensation. The strategy is to take up what Kaplan and Nelson[3] started, namely chiral perturbation theory (χ PT). Kaplan and Nelson predicted in tree order of χ PT that kaons condense in neutron matter at $\rho \lesssim 3\rho_0$. Our calculation goes to next-to-next-to-leading (NNL) order. It turns out that the calculation confirms the Kaplan-Nelson prediction although in the process new and interesting physical elements are uncovered. Our result is that for reasonable ranges of parameters involved, the critical density comes out to be

$$2 \lesssim \rho/\rho_0 \lesssim 4. \quad (1.1)$$

This is the range of densities relevant to the Bethe-Brown scenario for the formation of light-mass black holes for stars that exceed the critical mass of $M \simeq 1.56M_\odot$. Their arguments extend the estimated range of main sequence star masses, for which stars go into black holes, down to $\sim 18M_\odot$.

There are two situations where the production of kaons brings out interesting physics. One is their properties in relativistic heavy-ion collisions that involve temperature. Here kaon condensation is not directly relevant but the *mechanism* that triggers kaon condensation in the relevant situation has intriguing consequences on the properties of kaons observed in heavy-ion experiments. This is discussed in a recent review[4] and will not be discussed here. What we are interested in is what kaons do in cold dense matter appropriate to compact objects that result from the collapse of massive stars.

In stellar collapse, as matter density ρ increases, the electron chemical potential μ_e (determined by the chemical potentials of neutrons and protons in the system together with charge neutrality) increases, reaching several hundreds of MeV. If the electron chemical potential reaches the “effective mass” of a meson Φ , m_Φ , then the

electron can “decay” into a Φ as[5]

$$e^- \rightarrow \Phi^- + \nu_e. \quad (1.2)$$

In nature, the only low-mass bosons are the pseudo-Goldstone bosons $\Phi^- = \pi^-, K^-$. While lowest in mass, the pions do not seem to play an important role, so the next possible boson is the kaon with its mass ~ 500 MeV in free space. The electron chemical potential cannot reach this high, so on-shell kaons cannot be produced by this process. However as will be described below, the kaon in medium can undergo a mass shift due to density-dependent renormalization. As the μ_e increases and the effective kaon mass M_K^* decreases as ρ increases, the process (1.2) can occur at some density ρ_c . Kaons so produced will bose-condense at that density ρ_c . Whether or not this will occur then depends on whether or not M_K^* will decrease enough in density so that it meets μ_e . Such a condensation will be of physical interest if the critical density is low enough and the energy gain is high enough. This is the possibility we shall address below.

2 Maximum Neutron Star Mass

2.1 *Stellar Death Function*

That main sequence stars of mass $\geq 25 - 30M_\odot$ must end up in black holes without producing nucleosynthesis, i.e., without returning matter to the galaxy, is required by the observed abundances of elements[6, 7]. Maeder’s argument is based on the measurement of $\Delta Y/\Delta Z$, the ratio of helium abundance to that of metals, in low-metallicity extragalactic H_{II} regions, especially irregular dwarf galaxies. The ratio can be measured with good accuracy[6],

$$\frac{\Delta Y}{\Delta Z} = 4 \pm 1.3. \quad (2.3)$$

If all stable stars of mass up to $\sim 100M_\odot$ were to explode, returning matter to the galaxy, this ratio would lie between 1 and 2. Helium is produced chiefly by relatively light stars, metals by heavy stars, so that cutting off the production by the heavy stars going directly into black holes without nucleosynthesis increases the $\Delta Y/\Delta Z$. Using the standard initial mass function for stars,

$$dN/dM = M^{-(1+x)} \quad (2.4)$$

with $x = 1.35, 1.70$, Maeder[7] found that Pagel’s[8] measurement on $\Delta Y/\Delta Z$ was best reproduced by a cutoff of nucleosynthesis at a main sequence stellar mass of $\sim 22.5M_\odot$ as shown in Fig. 1.

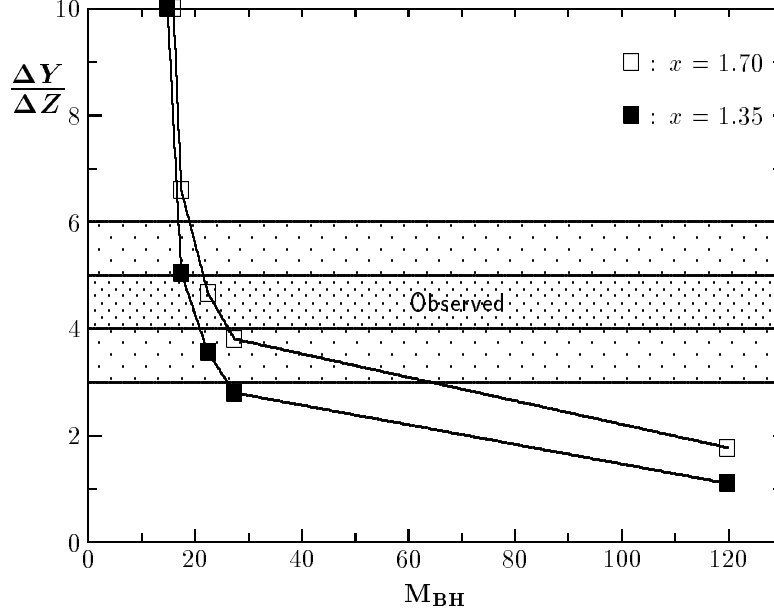


Figure 1: Values of the ratio $\Delta Y/\Delta Z$ of the relative helium to metal enrichments for different values of x as defined in eq.(2.4) as a function of M_{BH} . The data points correspond to initial metallicity $Z=0.001$. The observed range is indicated by shadings; the range 4 to 5 is the preferred one.[7]

There is considerable uncertainty in the initial mass function, as noted by Maeder[7], so that this limit could easily be $\sim 30M_{\odot}$ or even higher. Brown and Bethe estimated [1] $30M_{\odot}$ as the cutoff for stars to drop directly into black holes without nucleosynthesis.

What about stars with mass $20M_{\odot} < M < 30M_{\odot}$? Recent observation on SN1987A, whose progenitor mass is $\sim 18 \pm 2M_{\odot}$, gives us an insight. Based on the empirical analysis, Brown and Bethe also argued that a large range of stars below this mass, down to $\sim 18M_{\odot}$, can first accomplish nucleosynthesis and then collapse into black holes.

2.2 SN1987A: Neutron Star or Black Hole?

SN1987A(February 23, 1987) in the Large Magellanic Cloud is the nearest and brightest supernova to be observed since SN1604AD (Kepler), and certainly the most important supernova since SN1054AD, the progenitor of Crab Nebula. Because of its brightness and proximity, it will be possible to observe SN1987A for many years as it expands to reveal its inner secrets. In contrast, typical supernova, of which some 20-30 are observed each year, are some 1000 times further and 10^5 times fainter, and so become lost in their host galaxy within a year or two. Moreover, SN1987A has

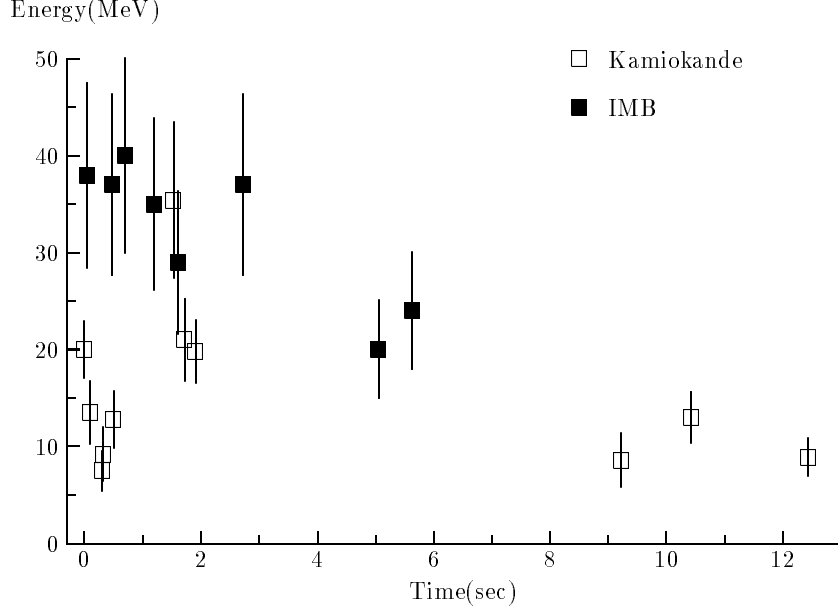
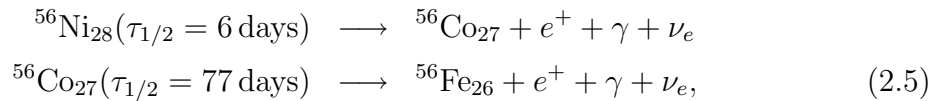


Figure 2: The energy of the neutrinos detected by the Kamiokande and IMB detectors is plotted as a single neutrino pulse. Most neutrinos arrived within the first second or two but there was a lower energy tail to the pulse which lasted more than 10 sec.

been observed at every wavelength band of the electromagnetic spectrum, from radio to γ -rays.

In supernova theory, the neutron star is followed by bursts of huge amount of neutrinos. In the case of SN1987A, neutrinos were detected by the IMB and Kamiokande detector about three hours before the optical burst. The total energy emitted in $\bar{\nu}_e$'s is $E_{\nu} \approx 3 \times 10^{52} \text{ erg/s}$, while the decay time scale of burst is $\Delta t \approx 10 \text{ sec}$. These are quite consistent with the expected values. Hence, firstly, it is believed that the neutron star was formed in SN1987A, even though there exist time gap of about 7 seconds between the eighth and ninth Kamiokande events, the gap being followed by another three events as shown in Fig. 2[9, 10].

From the observation of radioactive decay of ^{56}Ni and ^{56}Co ,



the mass of Ni ejecta of SN1987A is known to be $0.075 M_{\odot}$. Combining this result with the observed energy of SN1987A, $E = 1.4 \pm 0.4 \text{ foe}$ (where foe stands for 10^{51} erg), Brown and Bethe[1] obtain the range of the core mass of SN1987A

$$M = 1.535 \pm 0.02 M_{\odot}. \tag{2.6}$$

However, from the mass of the Hulse-Taylor pulsar, the lower limit of the compact core is known to be at least $1.44M_{\odot}$. Consequently, the compact core mass of SN1987A must range

$$1.44M_{\odot} < M_{core} < 1.56M_{\odot}. \quad (2.7)$$

Astronomers have been searching for a pulsar in the center of SN1987A remnants after the explosion. To see the explicit signal of a pulsar, one must wait until the remnants are transparent. If the pulsar is really formed in SN1987A, the X-ray signal should be detected within a few years after the explosion. The hypercritical accretion could hide the compact object for only ~ 1 year, but after this time, the 1987A should be observed with a luminosity $L = 4 \times 10^{38} \text{ ergs/s}$, but the present light curve is lower by two orders, $L \sim 4 \times 10^{36} \text{ ergs/s}$ [13]. This compact object, after being a proto-neutron star for at least 10 seconds, appears to have collapsed into a low-mass black hole. The core of SN1987A may have become a neutron star followed by neutrino emission, and later changed into low-mass black hole with mass about $1.5M_{\odot}$.

A possible scenario was proposed by Bethe. According to his arguments, a vigorous convection is produced in the supernova shock. But after about 2 seconds, the convection stops as heat is no longer supplied by neutrinos. As a result, a substantial fraction of the previously convecting material falls into the neutron star at the center. Bethe estimated this fraction to be about 10% of the mass in the shock wave, or about $0.04M_{\odot}$. If the neutron-star mass were close to M_{max} , the added $0.04M_{\odot}$ could push the neutron star over the limit and make it collapse into a black hole. If this happened in SN1987A, it must have been more than 12 seconds after the first collapse, since at 12 sec, a neutrino was still observed at Kamiokande II. A softened equation of state might be associated with a delayed collapse of the young neutron star into a black hole.

From standard evolutionary analysis, it is believed that the neutron star is formed if the core mass lies between the maximum neutron star mass (M_{max}) and the Chandrasekhar mass,[†]

$$M_{CH} = 5.76Y_e^2 M_{\odot}, \quad (2.8)$$

where Y_e is the electron fraction per baryon. In stellar collapse, Y_e is $0.43 \sim 0.50$.

[†] In the early 1930's, Chandrasekhar set a limit to the size of white dwarf. No carbon-rich white dwarf can support its weight if it is greater than about 1.4 times solar mass M_{\odot} . A massive star with its final mass after exhausting its fuel is greater than M_{CH} collapses and the collapse turns into an explosion.

Because of the thermal pressure, the evolutionary lower bound of the neutron star is about $(1.10 \sim 1.15) \times M_{CH} \approx (1.2 \sim 1.4)M_{\odot}$.

According to Brown and Bethe[1], in the binary pulsar evolution, the accretion can proceed at the hypercritical rate

$$\dot{M} \geq 10^4 \dot{M}_{Edd}, \quad (2.9)$$

where the Eddington limit is

$$\dot{M}_{Edd} = 1.5 \times 10^{-8} M_{\odot} yr^{-1}. \quad (2.10)$$

Hence, if the neutron star mass were determined by the evolutionary scenario, massive neutron stars with masses exceeding $1.5M_{\odot}$ should exist. But as shown below, none have been found.

2.3 *Observed Neutron Star Mass*

From radio, optical, x-ray and γ -ray surveys of supernova remnants, Helfand and Becker came to the conclusion that nearly half of the supernova in the galaxy leave no observable remnants. This is understandable because half of the supernova are Type I, which leave no neutron stars. However, as discussed in detail by Van den Bergh et al.[14], the observed samples of supernova in our galaxy are at low galactic latitude, so their light is strongly absorbed on its way to Earth. They argued that Type II supernova outnumber Type I supernova by a factor of several. Thus the conclusions of Helfand and Becker can be understood only if nearly half of the Type II supernova explosions do not form neutron stars. The possible candidate is, therefore, a low-mass black hole.

In Fig. 3, the measured neutron star masses are plotted. Most of all the neutron star masses are below $1.5M_{\odot}$ except for Vela X-1 and 4U 1700-37. But by the recent analysis[1], the mass of Vela X-1 is believed to be below $1.5M_{\odot}$. Further, there is an argument that 4U 1700-37 is a low-mass black hole[1]. If this is the case, it is striking that all well measured neutron star masses lie below $1.5M_{\odot}$. This calls for theoretical arguments to lower the maximum neutron star mass. In Table 1, the maximum allowed mass of a neutron star is given for various equations of state[15].

2.4 *Lowering Maximum Neutron Star Mass*

Since the pioneering work by Bahcall and Wolf[16], it has been known that a pion condensate, if it exists, will enhance strongly the emissivity of the neutrinos from

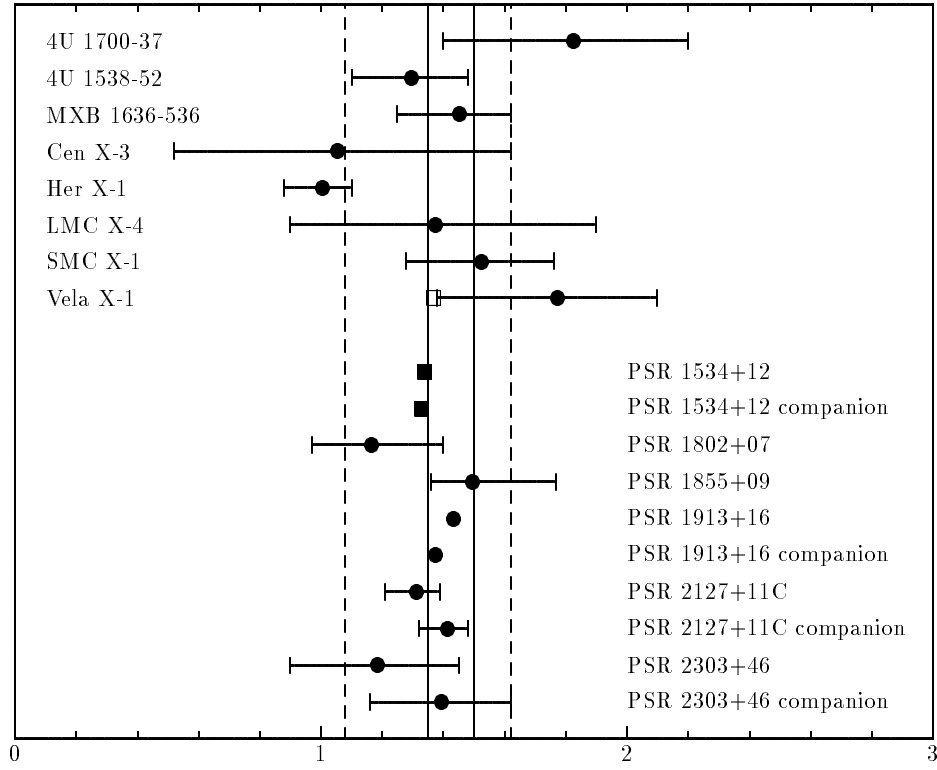


Figure 3: Neutron star masses in units of solar mass (M_{\odot}). The empty box at the lower end of Vela X-1 is given by the recent analysis of Van Kerkwijk.[1]

Equation of state	Maximum mass (M_{\odot})
π or K condensate	< 1.5
R	1.6
BJ	1.9
TNI	2.0
TI	2.0
MF	2.7

Table 1: The maximum mass of neutron star for various equations of state cited in [15]

within the core of a neutron star. These authors considered the reaction

$$n + \pi^- \rightarrow n + e^- + \nu \quad (2.11)$$

and its inverse reaction, in which the pions were treated as real particles existing with a certain probability inside the neutron star. Subsequently Maxwell et al[17] carried out a detailed calculation on the neutrino emissivity in the presence of pion condensates. The reaction mechanism may be written symbolically as

$$n + \text{“}\pi\text{”} \rightarrow n + e^- + \nu \quad (2.12)$$

where “ π ” represents the pion condensate built in the neutron quasiparticle states. The main conclusion of [17] was that even a small amount of pion condensates will cause a dramatic enhancement of neutrino emissivity making the equation of state softer than what standard calculations would predict. This soft equation of state could then lower the maximum neutron star mass.

However, the much-discussed P-wave pion condensation is now considered to be rather unlikely to take place at a low enough density. Based on renormalization-group flow equations, Lee et al[18] showed that the Yukawa coupling term responsible for P-wave condensation, $\bar{\psi}\vec{\tau} \cdot \vec{\pi}\gamma_5\psi$, becomes irrelevant after radiative corrections and cannot induce instability needed for a phase transition. Furthermore, the axial-vector coupling constant g_A in nuclear medium is effectively quenched, roughly, down to one.[19] The quenched axial-vector coupling would then push the critical density to a higher density ($> 5\rho_0$),

$$\rho_c \propto \frac{1}{g_A^2 - 1}, \quad (2.13)$$

where most of the approximations associated with effective hadronic Lagrangians must have broken down.

An S-wave pion condensation is also unlikely to occur since chiral symmetry protects the pion mass (PCAC). Even if S-wave condensation occurred, the effect would be negligible.

The next candidate process is kaon (K^-) condensation. (The kaon mass in free space is $\sim 495 MeV$). According to Kaplan and Nelson[3], the attraction needed for kaon condensation comes mainly from the KN sigma term

$$\delta M_K^2 \approx -\frac{\Sigma_{KN}}{f^2} \langle N^\dagger N \rangle + \dots \quad (2.14)$$

where Σ_{KN} comes from the explicit chiral symmetry breaking,

$$\Sigma_{KN} \simeq \frac{1}{2}(\hat{m} + m_s) \langle N | \bar{u}u + \bar{s}s | N \rangle. \quad (2.15)$$

Here \hat{m} and m_s are $\sim 5MeV$ and $\sim 150MeV$, respectively. The strangeness content of the proton is not well-determined, so that gives some uncertainty in the value of Σ_{KN} . The predicted critical density was found to be in the range

$$2.3\rho_0 \leq \rho_c \leq 3\rho_0. \quad (2.16)$$

Recently, an improved calculation was made by Thorsson et al.[20, 21]. However this calculation was also incomplete since the tree-order Lagrangian they used described correctly neither KN scattering nor kaonic atoms. In these talks, we describe how this defect is removed in chiral perturbation theory by going to one-loop order[22, 23, 24, 25].

3 Baryon Chiral Perturbation Theory

3.1 *Effective Field Theory for Nuclear Matter*

The process we are interested in requires a field theory that can describe simultaneously normal nuclear matter and phase transitions therefrom. The most relevant ingredient of QCD that is needed here is spontaneously broken chiral symmetry. We are specifically interested in chiral $SU(3) \times SU(3)$ symmetry since strangeness is involved. In order to address the problem, we need to start from a realistic effective chiral Lagrangian, obtain a nuclear matter of the right properties from it and then determine whether strangeness condensation occurs.

Unfortunately we do not yet know how to describe nuclear matter starting from a chiral Lagrangian. There are various suggestions and one promising one is that nuclear matter arises as a solitonic matter from a chiral effective action, a sort of chiral liquid[26] resembling Landau Fermi liquid. The hope is that the resulting effective action would look like Walecka's mean-field model. There is as yet no convincing derivation along this line. However as argued in [4], there is a compelling phenomenological indication that such a Fermi liquid structure can be identified with Walecka's mean field model provided that BR scaling is suitably implemented in the fluctuations. In the work reported here, we will have to assume that we have a nuclear matter that comes out of an effective chiral action. Given such a ground state containing no strange degrees of freedom, we would like to study fluctuations along the strangeness direction and determine if instability along that direction develops signaling a phase transition. We are therefore assuming that we can get the properties of normal nuclear matter from phenomenology, that is, that nuclear matter is a Fermi-liquid fixed point[27, 28]. In principle, a precise knowledge of this ground

state from a chiral effective Lagrangian at a nonperturbative QCD level would allow us to determine the coefficients that appear in the effective Lagrangian with which to describe fluctuations around the soliton background – i.e., the Fermi liquid – and with which we could then compute all nuclear response functions. At present such a derivation does not exist. In a recent paper by Brown and one of the authors (BR91)[29], it is assumed that in medium at a matter density $\rho \sim \rho_0$, the *nuclear* effective field theory can be written in terms of the medium-dependent coupling constants g^* and masses of hadrons m^* while preserving the free-space structure of a sigma model. This leads to the so-called Brown-Rho scaling. In BR91[29], the nonlinear sigma model implemented with trace anomaly of QCD is used to arrive at the scaling law. The precise way that this scaling makes sense is elaborated by Adami and Brown[30] and in the review (BR94)[4]. There have been numerous papers written with some of the essential points of this scaling misinterpreted.

Given such an effective field theory, we can make a general argument on the stability in various flavor directions of nuclear matter at high density. This can be done along the line of arguments developed for condensed matter physics by Shankar[27] and Polchinski[28] using renormalization group flow. We sketch the essential argument following Lee, Rho and Sin[18].

What we are interested in is whether the system in question develops instability along the direction of strangeness and if so, by which physical mechanism. This analysis will not give us the critical density. The critical density will be calculated by using chiral perturbation theory. For this purpose we will focus on the kaon frequency near the electron chemical potential. By Baym's theorem[31], one can identify the kaon chemical potential associated with charge conservation, μ_K , with the electron chemical potential, μ_e , which we shall simply write μ in what follows. This means that we will be looking at the vicinity of $\omega \sim \mu$ in the kaon dispersion formula. We shall assume that

$$|\omega - \mu| \ll \mu. \quad (3.17)$$

As mentioned, we assume that nucleons in nuclear matter are in Fermi-liquid state with the Fermi energy μ_F and the Fermi momentum k_F . Define ψ as the nucleon field fluctuating around the Fermi surface such that the momentum integral has a cut-off Λ_N ,

$$k_F - \Lambda_N < |\vec{k}| < k_F + \Lambda_N. \quad (3.18)$$

Kaons can interact with the nucleons through three-point functions of the KNN type (Yukawa interaction) and through four-point interactions of the $KKNN$ type. We

shall consider S-wave kaon-nucleon interactions, for which the Yukawa interaction can be ignored. A generic action involving the nucleon field ψ and the kaon field Φ can then be written, schematically, as

$$\begin{aligned}
S = & \int d\omega d^3q \Phi^*(\omega, \vec{q}) \left(\omega - q^2/2\mu_K \right) \Phi(\omega, \vec{q}) - \int d\omega d^3q \tilde{M}_K \Phi^* \Phi \\
& + \int (d\omega d^3q)^2 (d\epsilon d^3k)^2 h \Phi^* \Phi \psi^\dagger \psi \delta^4(\omega, \epsilon, \vec{q}, \vec{k}) \\
& + \int d\epsilon d^3k \psi^\dagger (\epsilon - \epsilon(k)) \psi + g \int (d\epsilon d^3k)^4 \psi^\dagger \psi^\dagger \psi \psi \delta^4(\epsilon, \vec{k}) \quad (3.19)
\end{aligned}$$

where $\tilde{M} = (M_K^2 - \mu^2)/2\mu$ and h and g are constants. The four-Fermi interaction with the coefficient g stands for Fermi-liquid interactions in nuclear matter. (In nuclear matter, one can have four such terms because of the nucleon spin and isospin degrees of freedom. We need not specify them for our purpose.) This is a toy action but it is generic in that the results of χ PT we will obtain below can be put into this form.

The renormalization group flow of this action can be analyzed in the following way. Since we are assuming that nuclear matter is a Fermi-liquid fixed point, fluctuations in the non-strange direction in the nucleon sector are stable: The four-Fermi interaction g is irrelevant or at best marginal. Fluctuations in the strange direction involve the kaon field Φ . Suppose we have integrated out all the high-frequency modes above the cut-off Λ measured with respect to μ . We are interested in the stability of the system under the renormalization group transformation $\Lambda \rightarrow s\Lambda$ ($s < 1$) as $s \rightarrow 0$. A scaling analysis shows that the interaction term h is irrelevant while the “mass term” \tilde{M} is relevant. The renormalization group-flow of the “mass term” and the interaction term h can be readily written down and solved[18] (with $t = -\ln s$),

$$\tilde{M}(t) = (\tilde{M}_0 - \frac{Dh_0}{1+a})e^t + \frac{Dh_0}{1+a}e^{-at} \quad (3.20)$$

with

$$h(t) = h_0 e^{-at}, \quad h_0 \geq 0 \quad (3.21)$$

where $D = \frac{3(1+\alpha^2)\alpha}{2\mu}\rho_N > 0$, $\alpha = \Lambda/k_F > 0$ and $a = 1/2$. We see from Eq.(3.20) that as $s \rightarrow 0$ for which $h \rightarrow 0$, \tilde{M} changes sign for some $(\tilde{M}_0, h_0 \geq 0)$. Thus although irrelevant, an attractive interaction h_0 determines the direction of the mass flow whereas it is the “mass term” that drives the system to instability.

3.2 Chiral Counting

Armed with the general information on the instability in the strangeness direction, we are now able to calculate the critical density in χ PT. As mentioned, we are to look at

the instability in the kaon direction, so it suffices for us to look at fluctuations around the Fermi-liquid state. For this we need an effective chiral Lagrangian involving baryons as well as Goldstone bosons. When baryons are present, χ PT is not as firmly formulated as when they are absent[32]. The reason is that the baryon mass m_B is $\sim \Lambda_\chi \sim 1$ GeV, the chiral symmetry breaking scale. It is more expedient, therefore, to redefine the baryon field so as to remove the mass from the baryon propagator

$$B_v = e^{im_B \gamma \cdot v \cdot x} P_+ B \quad (3.22)$$

where $P_+ = (1 + \gamma \cdot v)/2$ and write the baryon four-momentum

$$p_\mu = m_B v_\mu + k_\mu \quad (3.23)$$

where k_μ is the small residual momentum indicating the baryon being slightly off-shell. When acted on by a derivative, the baryon field B_v yields a term of $O(k)$. The (octet) baryon propagator simplifies in heavy-baryon formalism to $i/v \cdot k$ that involves no gamma matrices. This simplifies the loop calculation. The spin operator S_v^μ is defined by, $v \cdot S_v = 0$, $S_v^2 B_v = -\frac{3}{4} B_v$, $\{S_v^\mu, S_v^\nu\} = \frac{1}{2}(v^\mu v^\nu - g^{\mu\nu})$, and $[S_v^\mu, S_v^\nu] = i\epsilon^{\mu\nu\alpha\beta} v_\alpha (S_v)_\beta$. In the baryon rest frame, the spin operator S_v reduces to the usual spin operator $\vec{\sigma}/2$.

Chiral perturbation theory in terms of B_v and Goldstone bosons ($\pi \cdot \lambda/2$) is known as “heavy-baryon (HB) χ PT” [33]. HB χ PT consists of making chiral expansion in derivatives on Goldstone boson fields, ∂_M/Λ_χ , and on baryon fields, ∂_B/m_B , and in the quark mass matrix, $\kappa\mathcal{M}/\Lambda_\chi^2$. In the meson sector, this is just what Gasser and Leutwyler did for $\pi\pi$ scattering. In the baryon sector, consistency with this expansion requires that the chiral counting be made with $B^\dagger(\cdots)B$, not with $\bar{B}(\cdots)B$. This means that in medium, it is always the baryon density $\rho(r)$ that comes in and *not* the scalar density $\rho_s(r)$. This point seems to be misunderstood by some workers in the field.

Following Weinberg[34], we organize the chiral expansion in power Q^ν where Q is the characteristic energy/momentum scale we are looking at ($Q \ll \Lambda_\chi$) and

$$\nu = 4 - N_n - 2C + 2L + \sum_i \Delta_i \quad (3.24)$$

with the sum over i running over the vertices that appear in the graph and

$$\Delta_i = d_i + \frac{1}{2}n_i - 2. \quad (3.25)$$

Here ν gives the power of small momentum (or energy) for a process involving N_n nucleon lines, L number of loops, d_i number of derivatives (or powers of meson mass) in the i th vertex, n_i number of nucleon lines entering into i th vertex and C is the

number of separate connected pieces of the Feynman graph. Chiral invariance requires that $\Delta_i \geq 0$, so that the leading power is given by $L = 0$, $\nu = 4 - N_N - 2C$.

As an example, consider KN scattering. The leading term here is the tree graph with $\nu = 1$ and with $N_n = C = 1$. The next order terms are $\nu = 2$ tree graphs with $\Delta = 1$ that involves two derivatives or one factor of the mass matrix \mathcal{M} . From $\nu = 3$ on, we have loop graphs contributing together with appropriate counter terms.

In considering kaon-nuclear interactions as in the case of kaon condensation, we need to consider the case with $N_n \geq 2$ and $C \geq 2$. In dealing with many-body system, one can simply fix $4 - N_n$ and consider C explicitly. For instance if one has two nucleons (for reasons mentioned below, this is sufficient, with multinucleon interactions being suppressed), then we have $4 - N_n = 2$ but C can be 2 or 1, the former describing a kaon scattering on a single nucleon with a spectator nucleon propagating without interactions and the latter a kaon scattering irreducibly on a two-nucleon complex. Thus intrinsic n -nucleon processes are suppressed compared with $(n - 1)$ -nucleon processes by at least $O(Q^2)$. This observation will be used later for arguing that four-Fermi interactions are negligible in kaon condensation. This is somewhat like the suppression of three-body nuclear forces[34] and of three-body exchange currents[35] in chiral Lagrangians.

3.3 Kaon-Nucleon Scattering

Given a chiral Lagrangian, we need to first determine the parameters of the Lagrangian from available phenomenology. This is inevitable in effective field theories. We shall first look at kaon-nucleon scattering at low energies. This was done by Lee *et al.*[22, 24] which we summarize here. We shall compute the scattering amplitude to one-loop order and this entails a Lagrangian written to $O(Q^3)$ as one can see from the Weinberg counting rule. Instead of writing it out in its full glory, we write it in a schematic form as

$$\mathcal{L} = \sum_i \mathcal{L}_i[B_v, U, \mathcal{M}] \quad (3.26)$$

where the subscript i stands for ν relevant to the KN channel. Here B_v stands for both octet and decuplet baryons and U the Sugawara form for octet Goldstone bosons. For KN scattering in free-space, the Lagrangian is bilinear in the baryon field. Details are given in Lee *et al.* [22, 24]. Let us specify a few terms in (3.26) so as to streamline our discussion. Focusing on S-wave scattering, \mathcal{L}_1 contains the leading order term that may be described by the exchange of an ω between kaon and nucleon, attractive for K^-N and repulsive for K^+N and an isovector term corresponding to

the exchange of a ρ meson. These terms are proportional to the kaon frequency ω . To next order, \mathcal{L}_2 contains the “ KN sigma term” proportional to Σ_{KN}/f^2 where f is the pion decay constant and a term proportional to ω^2 which may be saturated by decuplet intermediate states. The $\nu = 3$ pieces are counter terms that contain terms that remove divergences in the loop calculations and finite terms that are to be determined from experiments. The complete S-wave scattering amplitudes calculated to the NNL order come out to be

$$\begin{aligned} a_0^{K^\pm p} &= \frac{m_B}{4\pi f^2(m_B + M_K)} \left[\mp M_K + (\bar{d}_s + \bar{d}_v)M_K^2 + \{(L_s + L_v) \pm (\bar{g}_s + \bar{g}_v)\} M_K^3 \right] \\ &\quad + \delta a_{\Lambda^*}^{K^\pm p} \\ a_0^{K^\pm n} &= \frac{m_B}{4\pi f^2(m_B + M_K)} \left[\mp \frac{1}{2} M_K + (\bar{d}_s - \bar{d}_v)M_K^2 + \{(L_s - L_v) \pm (\bar{g}_s - \bar{g}_v)\} M_K^3 \right] \end{aligned} \quad (3.27)$$

where M_K is the kaon mass, m_B the baryon (nucleon) mass, \bar{d}_s is the t-channel isoscalar contribution [$\mathcal{O}(Q^2)$], and \bar{d}_v is the t-channel isovector one [$\mathcal{O}(Q^2)$], both coming from \mathcal{L}_2 , $L_s(L_v)$ is the finite crossing-even t-channel isoscalar (isovector) finite one-loop contribution [$\mathcal{O}(Q^3)$] having the numerical values

$$L_s M_K \approx -0.109 \text{ fm}, \quad L_v M_K \approx +0.021 \text{ fm} \quad (3.28)$$

and the quantity $\bar{g}_s(\bar{g}_v)$ is the crossing-odd t-channel isoscalar (isovector) contribution [$\mathcal{O}(Q^3)$] from one-loop plus counter terms in \mathcal{L}_3 .

To understand the role of the Λ^* , we observe that the measured scattering lengths are repulsive in all channels except K^-n [36].

$$\begin{aligned} a_0^{K^+p} &= -0.31 \text{ fm}, & a_0^{K^-p} &= -0.67 + i0.63 \text{ fm} \\ a_0^{K^+n} &= -0.20 \text{ fm}, & a_0^{K^-n} &= +0.37 + i0.57 \text{ fm}. \end{aligned} \quad (3.29)$$

Although the experimental K^-N scattering lengths are given with error bars, the available K^+N data are not very well determined. Since both are used in fitting the parameters of the Lagrangian, we do not quote the error bars here and shall not use them for fine-tuning. For our purpose, we do not need great precision in the data as the results are extremely robust against changes in the parameters. The repulsion in K^-p scattering cannot be explained from Eq.(3.27) without the Λ^* contribution. In fact it is well known that the contribution of the $\Lambda(1405)$ bound state gives the repulsion required to fit empirical data for S-wave K^-p scattering [22, 23, 24, 37]. As mentioned, we may introduce the Λ^* as an elementary field. It takes the form

$$\delta a_{\Lambda^*}^{K^\pm p} = -\frac{m_B}{4\pi f^2(m_B + M_K)} \left[\frac{g_{\Lambda^*}^2 M_K^2}{m_B \mp M_K - m_{\Lambda^*}} \right] \quad (3.30)$$

which is completely determined given experimental data on the coupling g_{Λ^*} and the complex mass m_{Λ^*} .

There are four unknowns $\bar{d}_{s,v}$, $\bar{g}_{s,v}$ in (3.27) which can be determined from four experimental (real part of) scattering lengths Eq.(3.29). The results are

$$\begin{aligned}\bar{d}_s &\approx 0.201\text{fm}, & \bar{d}_v &\approx 0.013\text{fm}, \\ \bar{g}_s M_K &\approx 0.008\text{fm}, & \bar{g}_v M_K &\approx 0.002\text{fm}.\end{aligned}\tag{3.31}$$

So far, no prediction is made. However given the parameters so fixed, one can then go ahead and calculate the S-wave amplitude that enters in kaon condensation. This amounts to going off-shell in the ω variable, that is, in the kinematics where $\omega \neq M_K$. In doing this, one encounters an ambiguity due to the ω dependence of the coefficients \bar{d} which consist of the “ KN sigma term” and “ ω^2 term” which get compounded on-shell into one term. In the calculation reported in Lee *et al.*[22, 24], we chose to fix the “ ω^2 term” by resonance saturation and leave the “sigma term” to be fixed by the on-shell data. The predicted off-shell amplitudes[22, 24] agree reasonably with phenomenologically constructed off-shell amplitudes. All the constants of the chiral Lagrangian bilinear in the baryon field are thereby determined to $O(Q^3)$.

3.4 Four-Fermi Interactions

In medium, the chiral Lagrangian can have multi-Fermi interactions as a result of “mode elimination.” Here we consider four-Fermi interactions, ignoring higher-body interactions. We shall see that this is justified.

As stated above, we need to focus on four-Fermi interactions that involve strangeness degrees of freedom. Nonstrange four-Fermi interactions are subsumed in the Fermi-liquid structure of normal nuclear matter. For S-wave kaon-nuclear interactions, we only have the $\Lambda(1405)$ to account for. There are only two terms,

$$\mathcal{L}_{4\text{-fermion}} = C_{\Lambda^*}^S \bar{\Lambda}_v^* \Lambda_v^* \text{Tr } \bar{B}_v B_v + C_{\Lambda^*}^T \bar{\Lambda}_v^* \sigma^k \Lambda_v^* \text{Tr } \bar{B}_v \sigma^k B_v \tag{3.32}$$

where $C_{\Lambda^*}^{S,T}$ are the dimension -2 (M^{-2}) parameters to be fixed empirically and σ^k acts on baryon spinor. We shall now describe how to fix these two parameters from kaonic atom data.

In order to confront kaonic atom data, we need to calculate the kaon self-energy Π in nuclei. The off-shell amplitude determined above gives the so-called “impulse” term

$$\Pi_K^{imp}(\omega) = -\left(\rho_p \mathcal{T}_{free}^{K^-p}(\omega) + \rho_n \mathcal{T}_{free}^{K^-n}(\omega)\right) \tag{3.33}$$

where \mathcal{T}^{KN} is the off-shell S-wave KN transition matrix. (The amplitude \mathcal{T}^{KN} taken on-shell, *i.e.*, $\omega = M_K$, and the scattering length a^{KN} are related by $a^{KN} = \frac{1}{4\pi(1+M_K/m_B)}\mathcal{T}^{KN}$.) Medium corrections to this “impulse” term, obtained from one-loop graphs by replacing the free-space nucleon propagator by the in-medium propagator, shall be denoted as

$$- \left(\rho_p \delta \mathcal{T}_{\rho_N}^{K^-p}(\omega) + \rho_n \delta \mathcal{T}_{\rho_N}^{K^-n}(\omega) \right). \quad (3.34)$$

These two terms (3.33) and (3.34) are completely determined by the parameters fixed above. The new parameters of the four-Fermi interaction come into play in the first two self-energy graphs of Fig.4 (the last two graphs do not involve four-Fermi interactions but enter at the same order; they are free of unknown parameters),

$$\begin{aligned} \Pi_{\Lambda^*}(\omega) = & -\frac{g_{\Lambda^*}^2}{f^2} \left(\frac{\omega}{\omega + m_B - m_{\Lambda^*}} \right)^2 \left\{ C_{\Lambda^*}^S \rho_p \left(\rho_n + \frac{1}{2} \rho_p \right) - \frac{3}{2} C_{\Lambda^*}^T \rho_p^2 \right\} \\ & + \frac{g_{\Lambda^*}^2}{f^4} \rho_p \left(\frac{\omega}{\omega + m_B - m_{\Lambda^*}} \right) \omega^2 \left\{ (2\Sigma_K^p(\omega) + \Sigma_K^n(\omega)) \right. \\ & \left. - g_{\Lambda^*}^2 \left(\frac{\omega}{\omega + m_B - m_{\Lambda^*}} \right) (\Sigma_K^p(\omega) + \Sigma_K^n(\omega)) \right\} \end{aligned} \quad (3.35)$$

where g_{Λ^*} is the renormalized $KN\Lambda^*$ coupling constant determined in Lee *et al.*[22, 24] and $\Sigma_K^N(\omega)$ is a known integral that depends on proton and neutron densities and M_K . Note that while the second term of (3.35) gives repulsion corresponding to a Pauli quenching, the first term can give either attraction or repulsion depending on the sign of $(C_{\Lambda^*}^S[\rho_n + \frac{1}{2}\rho_p] - \frac{3}{2}C_{\Lambda^*}^T\rho_p)$. For symmetric nuclear matter, only the combination $(C_{\Lambda^*}^S - C_{\Lambda^*}^T)$ enters in the self-energy. This is an important element for kaonic atom.

The complete self-energy to in-medium two-loop order is then

$$\begin{aligned} \Pi_K(\omega) = & - \left(\rho_p \mathcal{T}_{free}^{K^-p}(\omega) + \rho_n \mathcal{T}_{free}^{K^-n}(\omega) \right) - \left(\rho_p \delta \mathcal{T}_{\rho_N}^{K^-p}(\omega) + \rho_n \delta \mathcal{T}_{\rho_N}^{K^-n}(\omega) \right) \\ & + \Pi_{\Lambda^*}(\omega). \end{aligned} \quad (3.36)$$

We now turn to fixing the constants of the four-Fermi interactions based on the recent analysis of kaonic atoms by Friedman, Gal and Batty [38]. For later purpose we shall parameterize the proton and neutron densities by the proton fraction x and the nucleon density $u = \rho/\rho_0$ as

$$\rho_p = x\rho, \quad \rho_n = (1-x)\rho, \quad \rho = u\rho_0. \quad (3.37)$$

Now Friedman *et al.*[38] found from their analysis that the optical potential for the K^- in medium has an attraction of the order of

$$\Delta V \equiv M_K^* - M_K \approx -(200 \pm 20) \text{ MeV at } u = 0.97 \quad (3.38)$$

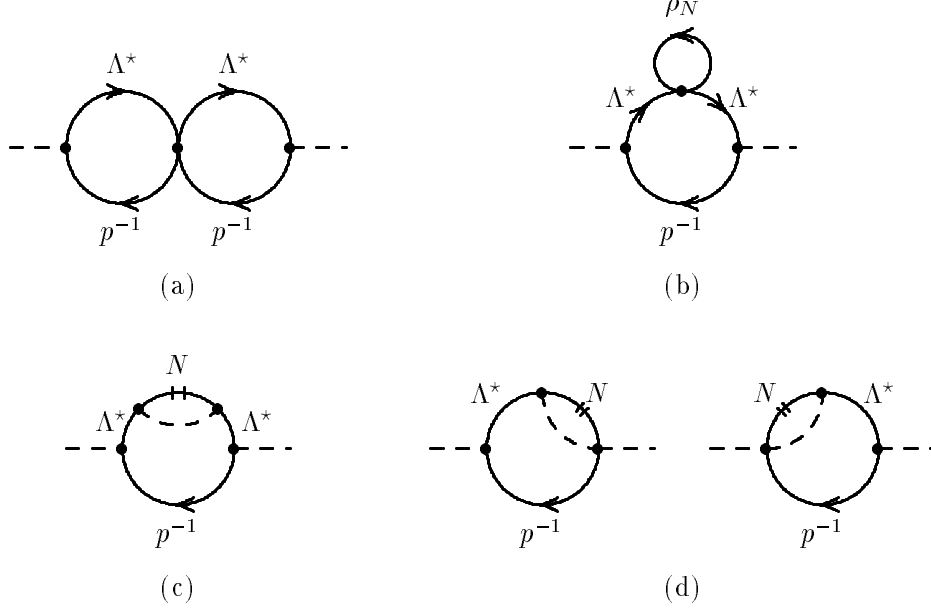


Figure 4: The in-medium two-loop kaon self-energy involving $\Lambda(1405)$. Figures a and b contain the constants of the four-Fermi interaction and figures c and d are Pauli corrections

with

$$M_K^* \equiv \sqrt{M_K^2 + \Pi_K}. \quad (3.39)$$

This implies approximately for $x = 1/2$

$$(C_{\Lambda^*}^S - C_{\Lambda^*}^T)f^2 \approx 20. \quad (3.40)$$

Friedman *et al.* [38] note that their “nominal” optical potential gives an attraction of order of 800 MeV when extrapolated to three times the normal density. We show in Table 2 what our theory predicts at higher densities than normal.[‡] At $u = 3$, the net attraction is only about 1.7 times the one at $u = 1$.

Equation (3.35) shows that for symmetric nuclear matter ($x = 1/2$), the combination $(C_{\Lambda^*}^S + C_{\Lambda^*}^T)$ does not enter into the self-energy formula. In order to extract it as needed for non-symmetric system as in compact star matter, we need information

[‡] The numerical values in Tables 2 and 3 are slightly modified from the previous results in [25] which had numerically small errors. Specifically, the modifications in $\delta\mathcal{T}_{\rho_N}^{K^-N}$, Eq. (G.3) of [24],

$$D_{6,ij}^N \longrightarrow -D_{6,ij}^N, \quad M_i^2 \Sigma_i^N(0) \longrightarrow \frac{1}{2\pi^2} \int_0^{k_{FN}} d|\vec{k}| \frac{|\vec{k}|^4}{M_i^2 + |\vec{k}|^2},$$

are responsible for the slight changes in the numerical values. We have verified that our new results (See Eq. (G.1) of [24]) satisfy the chiral symmetry constraint of Meißner et al.[39].

u	M_K^*	ΔV
0.5	354.9	-140.1
1.0	294.4	-200.6
1.5	249.2	-245.8
2.0	211.7	-283.3
2.5	180.5	-314.5
3.0	153.9	-341.1
3.5	130.8	-364.2
4.0	113.2	-381.8

Table 2: K^- effective mass(M_K^*) and the attraction ($\Delta V \equiv M_K^* - M_K$) in symmetric nuclear matter ($x = 0.5$) as function of density u in unit of MeV for $(C_{\Lambda^*}^S - C_{\Lambda^*}^T)f^2 = 20$.

for nuclei with $x \neq 1/2$. This can be done from the results of Friedman *et al.* by noting that our self-energy is nonlinear in x , so

$$\frac{\partial \Delta V}{\partial x}(C_{\Lambda^*}^S, \rho \approx \rho_0)|_{x=1/2} \approx 400 \, b_1/b_0 \, \text{MeV} \quad (3.41)$$

where $b_{0,1}$ are the constants given by Friedman *et al.*. This relation determines the coefficient $C_{\Lambda^*}^S$. The result is shown in Table 3 (first three columns).

Friedman *et al.*[38] find the acceptable value to be $b_1/b_0 = -0.56 \pm 0.82$. But there is one point which needs to be discussed in interpreting this number in the context of our theory. The constant $C_{\Lambda^*}^S$ shifts linearly the effective in-medium mass of $\Lambda(1405)$, with the mass shift being given by

$$\delta m_{\Lambda^*} = \sum_{i=a,b} \delta \Sigma_{\Lambda^*}^{(i)}(\omega = m_{\Lambda^*} - m_B) \quad (3.42)$$

where

$$\begin{aligned} \delta \Sigma_{\Lambda^*}^{(a)}(\omega) &= -\frac{g_{\Lambda^*}^2}{f^2} \omega^2 (\Sigma_K^p(\omega) + \Sigma_K^n(\omega)) \\ \delta \Sigma_{\Lambda^*}^{(b)}(\omega) &= -C_{\Lambda^*}^S (\rho_p + \rho_n). \end{aligned} \quad (3.43)$$

For nuclear matter density $u = 1$ and $x = 1/2$, the shift is

$$\delta m_{\Lambda^*}(u, x, y) \approx [62 - 150.3 \times y] \, \text{MeV} \quad (3.44)$$

with $y = C_{\Lambda^*}^S f^2$. It seems highly unlikely that the $\Lambda(1405)$ will be shifted by hundreds of MeV in nuclear matter. This means that y must be of $O(1)$, and *not* $O(10)$. For

$y = C_{\Lambda^*}^S f^2$	$\partial\Delta V/\partial x$	b_1/b_0	u_c		
			$F(u) = \frac{2u^2}{1+u}$	$F(u) = u$	$F(u) = \sqrt{u}$
50	125.44MeV	0.314	2.25	2.50	2.97
40	64.77MeV	0.162	2.33	2.58	3.08
30	4.10MeV	0.010	2.42	2.69	3.22
20	-56.58MeV	-0.141	2.54	2.84	3.41
10	-117.35MeV	-0.293	2.71	3.05	3.71
0.41	-175.43MeV	-0.439	2.98	3.43	4.28
0	-177.92MeV	-0.445	2.99	3.45	4.32
-10	-238.59MeV	-0.596	3.60	4.85	~ 6.41

Table 3: Determination of $C_{\Lambda^*}^S$ from the kaonic atom data[38] and the critical density (obtained with the constant so determined) for kaon condensation for various forms of symmetry energy $F(u)$ and $(C_{\Lambda^*}^S - C_{\Lambda^*}^T)f^2 = 20$. $y = 0.41$ corresponds to no $\Lambda(1405)$ mass shift in medium at the normal matter density.

$y = 0.41$ which corresponds to $b_1/b_0 \approx -0.4$, there is no shift at normal matter density. We believe this is a reasonable value. In fact, $y = 0$ is also acceptable. It would be interesting to measure the shift of $\Lambda(1405)$ to fix the constant $C_{\Lambda^*}^S$ more precisely although its precise magnitude seems to matter only a little for kaonic atoms and as it turns out, negligibly for kaon condensation.

Let us comment briefly on the role of multi-Fermion Lagrangians. The Weinberg counting rule shows that the four-Fermi interactions are suppressed by $O(Q^2)$ relative to the terms involving bilinears of Fermi fields. In general n -Fermi interactions will be suppressed by the same order relative to $(n-1)$ -Fermi interactions. In considering kaon condensation, what this means in conjunction with the renormalization-group flow argument, is that n -Fermi interactions with $n \geq 4$ are irrelevant in the RGE sense, and hence unimportant for condensation. The situation with the kaonic atom data is a bit different. While the strength of the four-Fermi interaction, y , is not important (this can be seen in Lee *et al.*[24], Table 3), its presence is essential for the attraction that seems to be required. This is in contrast to the kaon condensation which is driven by the “mass flow” with four-Fermi interactions being irrelevant in the RGE sense.

4 Kaon Condensation

4.1 Equation of State and Critical Densities

We have now all the ingredients needed to calculate the critical density for negatively charged kaon condensation in dense nuclear star matter. For this, we will follow the procedure given in work of Thorsson, Prakash and Lattimer (TPL)[21]. As argued by Brown, Kubodera and Rho[5], we need not consider pions when electrons with high chemical potential can trigger condensation through the process $e^- \rightarrow K^- \nu_e$. Thus we can focus on the spatially uniform condensate

$$\langle K^- \rangle = v_K e^{-i\mu t}. \quad (4.45)$$

The energy density $\tilde{\epsilon}$ – which is related to the effective potential in the standard way – is given by,

$$\begin{aligned} \tilde{\epsilon}(u, x, \mu, v_K) = & \frac{3}{5} E_F^{(0)} u^{\frac{5}{3}} \rho_0 + V(u) + u \rho_0 (1 - 2x)^2 S(u) \\ & - [\mu^2 - M_K^2 - \Pi_K(\mu, u, x)] v_K^2 + \sum_{n \geq 2} a_n(\mu, u, x) v_K^n \\ & + \mu u \rho_0 x + \tilde{\epsilon}_e + \theta(|\mu| - m_\mu) \tilde{\epsilon}_\mu \end{aligned} \quad (4.46)$$

where $E_F^{(0)} = (p_F^{(0)})^2 / 2m_B$ and $p_F^{(0)} = (3\pi^2 \rho_0 / 2)^{\frac{1}{3}}$ are, respectively, Fermi energy and momentum at nuclear density. The $V(u)$ is a potential for symmetric nuclear matter as described by Prakash *et al.*[40] which is presumably subsumed in contact four-Fermi interactions (and one-pion-exchange – nonlocal – interaction) in the non-strange sector as mentioned above. It will affect the equation of state in the condensed phase but not the critical density, so we will drop it from now on. The nuclear symmetry energy $S(u)$ – also subsumed in four-Fermi interactions in the non-strange sector – does play a role as we know from Prakash *et al.*[40]: Protons enter to neutralize the charge of condensing K^- 's making the resulting compact star “nuclear” rather than neutron star as one learns in standard astrophysics textbooks. We take the form advocated by Prakash *et al.*[40]

$$S(u) = \left(2^{\frac{2}{3}} - 1\right) \frac{3}{5} E_F^{(0)} \left(u^{\frac{2}{3}} - F(u)\right) + S_0 F(u) \quad (4.47)$$

where $F(u)$ is the potential contributions to the symmetry energy and $S_0 \simeq 30 \text{ MeV}$ is the bulk symmetry energy parameter. We use three different forms of $F(u)$ [40]

$$F(u) = u, \quad F(u) = \frac{2u^2}{1+u}, \quad F(u) = \sqrt{u}. \quad (4.48)$$

The contributions of the filled Fermi seas of electrons and muons are[21]

$$\begin{aligned} \tilde{\epsilon}_e &= -\frac{\mu^4}{12\pi^2} \\ \tilde{\epsilon}_\mu &= \epsilon_\mu - \mu \rho_\mu = \frac{m_\mu^4}{8\pi^2} \left((2t^2 + 1)t\sqrt{t^2 + 1} - \ln(t^2 + \sqrt{t^2 + 1}) \right) - \mu \frac{p_{F_\mu}^3}{3\pi^2} \end{aligned} \quad (4.49)$$

$g_{\Lambda^*}^2$	$(C_{\Lambda^*}^S - C_{\Lambda^*}^T)f^2$	$C_{\Lambda^*}^S f^2 = 100$	$C_{\Lambda^*}^S f^2 = 10$	$C_{\Lambda^*}^S f^2 = 0$
0.25	1	2.25	3.29	4.91
	10	2.25	3.16	3.76
	100	2.18	2.67	2.79

Table 4: Critical density u_c in in-medium two-loop chiral perturbation theory for $F(u) = u$.

where $p_{F_\mu} = \sqrt{\mu^2 - m_\mu^2}$ is the Fermi momentum and $t = p_{F_\mu}/m_\mu$.

The ground-state energy prior to kaon condensation is then obtained by extremizing the energy density $\tilde{\epsilon}$ with respect to x , μ and v_K :

$$\left. \frac{\partial \epsilon}{\partial x} \right|_{v_K=0} = 0, \quad \left. \frac{\partial \epsilon}{\partial \mu} \right|_{v_K=0} = 0, \quad \left. \frac{\partial \epsilon}{\partial v_K^2} \right|_{v_K=0} = 0 \quad (4.50)$$

from which we obtain three equations corresponding, respectively, to beta equilibrium, charge neutrality and dispersion relation. The critical density so obtained is given for three different $F(u)$'s in Table 3, and for various ranges of parameters in Table 4. The result is

$$2 < u_c \lesssim 4. \quad (4.51)$$

We note that the largest sensitivity is associated with the part that is not controlled by chiral symmetry, namely the density dependence of the symmetry energy function $F(u)$. This uncertainty reflects the part of interaction that is not directly given by chiral Lagrangians, that is, the part leading to normal nuclear matter. This is the major short-coming of our calculation.

Related to this issue is BR scaling. As we argued, were we able to derive nuclear matter from effective chiral Lagrangians, we would have parameters of the theory determined at that point reflecting the background around which fluctuations are to be made. The BR scaling was proposed in that spirit but with a rather strong assumption: That a sigma model governs dynamics in medium as in free space with only coupling constants and masses scaled a function of density. Up to date, no derivation of this scaling from basic principles has been made. In this sense, we might consider it as a conjecture although there is strong support for it from Walecka phenomenology in mean field as discussed in [4]. Suppose we apply BR scaling. The only way the procedure can make sense is to apply the scaling argument to the tree order terms, but not to the loop corrections. The result of this procedure is significant in that the critical density is brought down in an intuitively plausible way to about

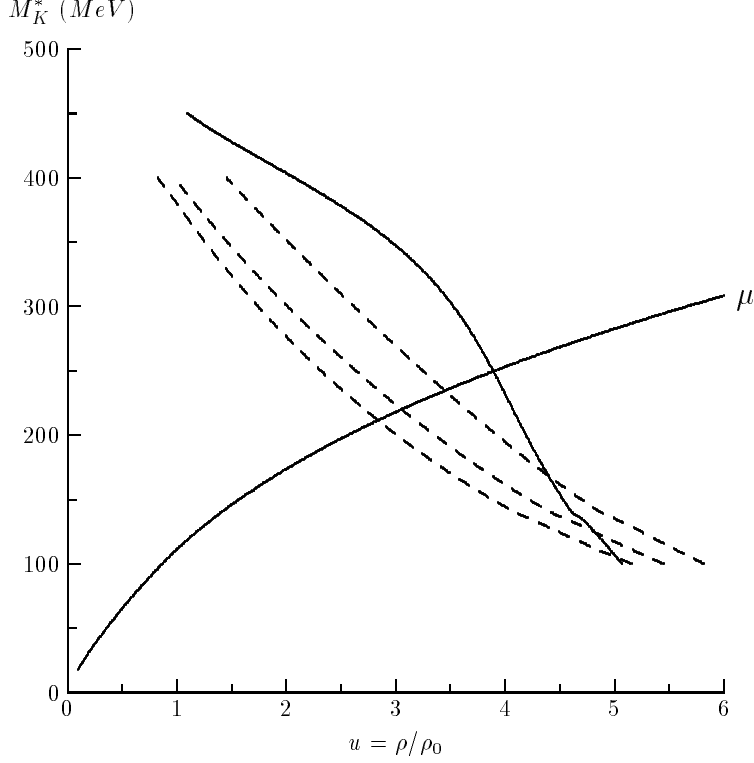


Figure 5: Plot of the quantity M_K^* obtained from the dispersion formula $D^{-1}(\mu, u) = 0$ vs. the chemical potential μ prior to kaon condensation for $g_{\Lambda^*}^2 = 0.25$ and $F(u) = u$. The solid line corresponds to impulse approximation and the dashed lines to the in-medium two-loop results for $(C_{\Lambda^*}^S - C_{\Lambda^*}^T)f^2 = 20$ and $C_{\Lambda^*}^S f^2 = 20, 10, 0$ respectively from the left. The point at which the chemical potential μ intersects M_K^* corresponds to the critical point.

$u_c \sim 2$, with very little dependence on parameters, loop corrections and multi-Fermi interactions. Thus slightly modified from (4.51), we arrive at the announced result

$$2 \lesssim u_c \lesssim 4. \quad (4.52)$$

4.2 Irrelevance of Λ^* to Kaon Condensation

To see which modes are involved in S-wave kaon condensation, we consider the dispersion formula at tree order,

$$D^{-1}(\omega) = \omega^2 - M_K^2 - \Pi(\omega). \quad (4.53)$$

As shown in Fig. 6 by the solid line, the kaon “effective mass” M_K^* is reduced mainly by the KN sigma term when there are no Λ^* contributions. If we turn on the Λ^*

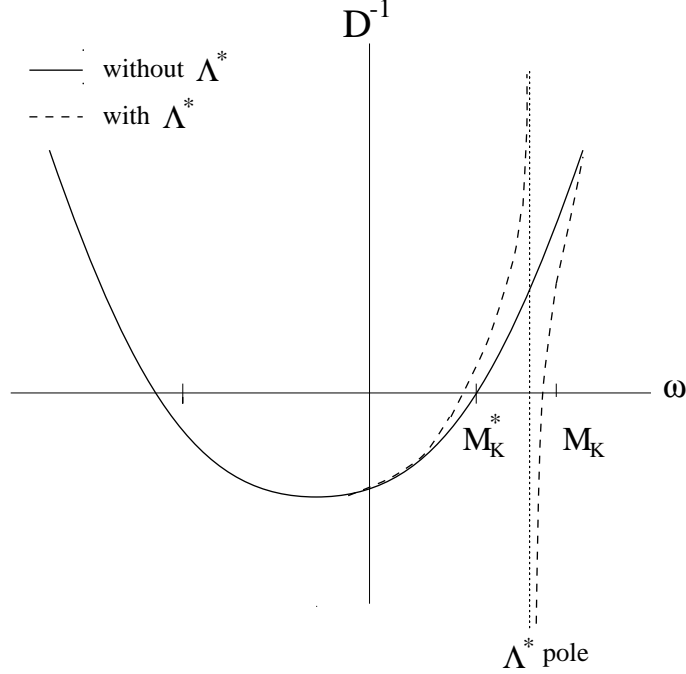


Figure 6: Plot of $D^{-1} = \mu^2 - M_K^2 - \Pi(\mu)$

coupling, there will be additional attractions. However since the effective mass M_K^* lies far from the Λ^* -pole contribution, the resulting magnitude of the attraction is small, i.e., the M_K^* remains nearly unmodified. Furthermore, since the Λ^* -pole is far outside of M_K^* , the condensed kaon mode remains the same independently of the Λ^* .

Summarizing the results, Λ^* may be crucial for understanding the KN scattering and kaonic atom data, but is irrelevant to determining the kaon condensation. The critical densities for wide ranges of the Λ^* coupling in Table 4 confirm the unimportance of Λ^* contribution.

5 Kaons on the Hypersphere

Recently, using the idea of Manton[41] for simulating density effects, Westerberg[42] explored S-wave kaon condensation in the bound-state approach to the Skyrme model on a 3-sphere. The spatial metric in a hypersphere of radius a is

$$ds^2 = a^2(d\rho^2 + \sin^2 \rho(d\theta^2 + \sin^2 \theta d\phi^2)) \quad (5.54)$$

where the possible ranges of three angular coordinates are

$$0 \leq \rho, \quad \theta \leq \pi, \quad 0 \leq \phi \leq 2\pi. \quad (5.55)$$

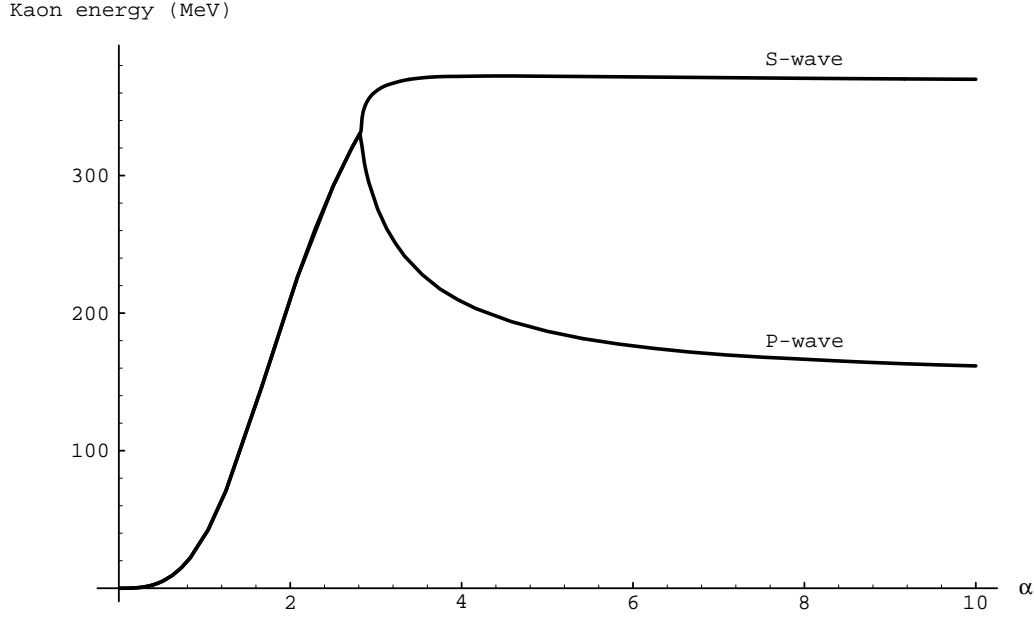


Figure 7: Kaon energy (MeV) vs. α [42].

The baryon number density is given by the inverse volume of the hypersphere

$$\rho = \frac{1}{2\pi^2 a^3} \equiv \frac{e^3 F_\pi^3}{2\pi^2 \alpha^3} \quad (5.56)$$

where $\alpha = aeF_\pi$ with $F_\pi = 129\text{MeV}$ and $e = 5.45$. The kaon energy is shown in Fig. 7 which also shows that the chiral phase transition occurs at $\alpha = \alpha_c = 2\sqrt{2}$. For $\alpha > \alpha_c$, the P-wave and S-wave kaons have different masses, the difference in mass representing roughly the mass difference between $\Lambda(1405)$ and $\Lambda(1116)$. For $\alpha < \alpha_c$, the S-wave and P-wave kaons become degenerate, with the kaon condensation occurring in the regime $1 < \alpha < 2\sqrt{2}$.

Solving the equation of state for the electron chemical potential, Westerberg[42] found the critical density to be at $\alpha = 1.58$ corresponding to $\rho \approx 3.7\rho_0$, Eq.(5.56). This falls within our predicted range $(2 \sim 4)\rho_0$. However an unsatisfactory aspect of this result is that the kaon condensation sets in *after* – and *not* before as one expects – the chiral phase transition.

6 Discussion

Introducing Λ^* up to order Q^3 , we obtain the critical density close to that of Kaplan and Nelson[3], and confirm that the KN sigma term, as in original approach of [3], is essential for kaon condensation. This result is further supported by a renormalization-group flow argument as well as by the recent bound-state approach to the Skyrme

model.

Given that kaon condensation occurs at a low enough density as predicted here, the Bethe-Brown scenario seems very plausible. However whether or not the Bethe-Brown scenario[1] of compact star formation is fully supported by the chiral Lagrangian approach will have to await the calculation of the equation of state at in-medium two-loop order, which is in progress. Our conjecture is that to the extent that our work confirms the original Kaplan-Nelson calculation[3], the compact star properties calculated previously at the tree level[21] would come out qualitatively unmodified in the higher-order chiral perturbation theory.

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